



## Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact [support@jstor.org](mailto:support@jstor.org).

**353. Proposed by CLIFFORD N. MILLS, Brookings, North Dakota.**

A uniform beam of oak, 10 feet long, 15 inches deep and 10 inches wide, sustains, in addition to its own weight, a load of 5,000 lbs. placed at the center. Find the greatest bending moment and the greatest stress in the fibers. Take the specific gravity of oak as .934.

**NUMBER THEORY.****270. Proposed by GERSHOM N. CARMICHAEL, Urbana, Ill.**

Does there exist a fraction  $p/q$  in its lowest terms such that the ratio of the sum of the divisors of  $p$  to the sum of the divisors of  $q$  is equal to  $p/q$ ? Give a method of finding such fractions not in their lowest terms.

**271. Proposed by HORACE OLSON, Chicago, Ill.**

Prove that if  $x, y, z, u, v$ , and  $w$  are integers such that  $x^2 + y^2 = u^2, x^2 + z^2 = v^2, y^2 + z^2 = w^2$ , then the product  $xyzuvw$  is divisible by 518400.

**SOLUTIONS OF PROBLEMS.****ALGEBRA.****474. Proposed by A. A. BENNETT, University of Texas.**

Show that the value of the infinite continued fraction, all of whose coefficients are unity,

$$1 + \frac{1}{1 + \frac{1}{1 + \dots}}, \text{ is } \frac{1}{2}(1 + \sqrt{5}).$$

Also find an explicit algebraic formula for the  $n$ th convergent.

SOLUTION BY C. C. YEN, Tangshan, North China.

1. Let  $x$  denote the value of the continued fraction. Then,

$$x = 1 + 1/x, \text{ i. e., } x^2 - x - 1 = 0,$$

therefore,

$$x = \frac{1}{2}(1 + \sqrt{5}),$$

where the sign for the radical is positive, since  $x$  is evidently positive.

2. Let  $p_n/q_n$  denote the  $n$ th convergent. Then

$$p_1 = 1, \quad q_1 = 1, \quad p_2 = 2, \quad q_2 = 1. \quad (\text{I})$$

Also, since all the coefficients are unity,

$$p_n = p_{n-1} + p_{n-2}, \quad q_n = q_{n-1} + q_{n-2} \quad (n = 3, 4, 5, \dots). \quad (\text{II})$$

And, it follows, therefore, from I and II,

$$q_n = p_{n-1} \quad (n = 2, 3, 4, \dots). \quad (\text{III})$$

Now consider the series

$$1 + x + 2x^2 + p_3x^3 + p_4x^4 + \dots + p_nx^n + \dots,$$

where the coefficients  $p_n$  satisfy II. It is a recurring series whose scale of relation is  $1 - x - x^2$ , and whose generating function is  $1/(1 - x - x^2)$ . Hence,  $p_n$  is the coefficient of  $x^n$  of the expansion of this function; and, by III,  $q_n$  is the coefficient of  $x^{n-1}$  of the same expansion.

If we put  $\alpha = -\frac{1}{2}(1 + \sqrt{5})$ ,  $\beta = -\frac{1}{2}(1 - \sqrt{5})$ , we get

$$\frac{1}{1 - x - x^2} = \frac{1}{\alpha - \beta} \left( \frac{1}{\alpha - x} - \frac{1}{\beta - x} \right),$$

whence the general term of the expansion of  $1/(1 - x - x^2)$  is

$$\frac{1}{\alpha - \beta} \left\{ \left( \frac{1}{\alpha} \right)^{n+1} - \left( \frac{1}{\beta} \right)^{n+1} \right\} x^n = \frac{1}{\alpha - \beta} \left\{ \frac{\beta^{n+1} - \alpha^{n+1}}{(\alpha\beta)^{n+1}} \right\} x^n.$$

But  $\alpha\beta = -1$ , and  $q_n = p_{n-1}$ . It follows, therefore, that

$$p_n = (-1)^n \left\{ \frac{\alpha^{n+1} - \beta^{n+1}}{\alpha - \beta} \right\}, \quad q_n = (-1)^{n-1} \left\{ \frac{\alpha^n - \beta^n}{\alpha - \beta} \right\}.$$

Hence, finally, the  $n$ th convergent is given by

$$-(\alpha^{n+1} - \beta^{n+1})/(\alpha^n - \beta^n),$$

where  $\alpha, \beta$  are the roots of the equation  $x^2 - x - 1 = 0$ .

Also solved by PAUL CAPRON, N. P. PANDYA, and O. S. ADAMS.

**475. Proposed by E. B. ESCOTT, Kansas City, Mo.**

A man makes a contract to purchase a house, making a cash payment down and agreeing to make monthly payments of  $a$  dollars, interest being charged at 6 per cent., the balance of the monthly payments being credited on the principal. Find a formula for  $M_n$ , the balance due after  $n$  payments.

SOLUTION BY C. R. DUNCAN, Amherst, Massachusetts.

Let  $M_0$  = balance due after the original cash payment, and  $r$  = rate of interest per month ( $= .06/12$ ), then  $M_0r$  = interest due at end of first month, and  $a - M_0r$  = amount paid back on the principal  $M_0$ .

At the end of the second month the interest would be less, the difference being equal to the interest on the amount paid back the first month, or  $(a - M_0r)r$ . But as all monthly payments are to be equal this amount would be credited on the principal. Hence, the amount paid back on the principal at the end of the second month is

$$(a - M_0r) + (a - M_0r)r = (a - M_0r)(1 + r).$$

Similarly, the amount paid back at the end of the third month would be equal to the amount paid back the second month plus the difference in interest between the second and third months, or

$$(a - M_0r)(1 + r) + (a - M_0r)(1 + r)r = (a - M_0r)(1 + r)^2.$$

Hence, the amount paid back at the end of the  $n$ th month  $= (a - M_0r)(1 + r)^{n-1}$ . Therefore, the total amount paid back in  $n$  months is

$$\begin{aligned} (a - M_0r) + (a - M_0r)(1 + r) + (a - M_0r)(1 + r)^2 + \cdots + (a - M_0r)(1 + r)^{n-1} \\ = \frac{(a - M_0r)[(1 + r)^n - 1]}{r}, \end{aligned}$$

and the balance due after  $n$  payments is

$$M_n = M_0 - \frac{(a - M_0r)[(1 + r)^n - 1]}{r}.$$

Putting the right-hand member equal to 0 and solving for  $a$ , we have a formula for finding the amount of the monthly payments required to pay back the principal  $M_0$  in a given number of months,

$$a = \frac{M_0r(1 + r)^n}{(1 + r)^n - 1}.$$

Also solved by G. W. HARTWELL, E. J. OGLESBY, HORACE OLSON, A. R. NAUER, PAUL CAPRON, G. PAASWELL, H. N. CARLETON, J. B. REYNOLDS, and the PROPOSER.

**476. Proposed by W. HAROLD WILSON, University of Illinois.**

Prove that if  $x_h \neq x_j$

$$(h, j = 1, 2, \dots, n, h \neq j),$$

then

$$\sum_{i=1}^n \frac{x_i^{n-1}}{\prod'_{h=1} (x_i - x_h)} = 1,$$

where the prime indicates the omission of zero factors in the denominator.